Title
Proton β decay in large magnetic fields

Journal
Physics Letters B, 311(1-4)

ISSN
0370-2693

Authors
Bander, M
Rubinstein, HR

Publication Date
1993-07-29

DOI
10.1016/0370-2693(93)90553-T

Physics Letters B 311 (1993) 187–192
North-Holland

PHYSICS LETTERS B

Proton β decay in large magnetic fields

Myron Bander [1]

Department of Physics, University of California, Irvine, CA 92717, USA

and

H.R. Rubinstein [2]

Department of Radiation Sciences, University of Uppsala, Uppsala, Sweden

Received 26 April 1992; revised manuscript received 23 April 1993
Editor: H. Georgi

Both electromagnetic and strong interactions contribute to make, in magnetic fields of the order of $B > 10^{14}$ T, the neutron stable against beta decay and for somewhat larger fields the proton becomes unstable to a decay into a neutron via β emission. Changes in chiral condensates due to such fields and an unexpectedly large field dependence of hadronic magnetic moments would modify these arguments. Fields of such magnitude may exist in colliding neutron stars and in the vicinity of cosmic strings. Possible astrophysical consequences are discussed.

1. Introduction

The studies of the stability of the standard model of particle physics in the presence of intense magnetic fields was initiated by Ambjørn and Olesen [1]. These authors find that instabilities will occur for fields larger than M_W^2/e or $\sim 10^{20}$ T. Fields, though intense but several orders of magnitude smaller, have been postulated to exist near astrophysical objects [2,3]; we have been investigating [4,5] questions related to the behavior of elementary and composite states in this environment. In this paper we concentrate on the question of mass shifts of bound states of quarks. These shifts occur both due to the effects such magnetic fields have on the strong binding forces, and due to the direct interactions of charged spinning particles with external fields. The modifications of the strong forces are such as to close the gap between the proton and neutron masses and ultimately make the proton heavier; a delicate interplay between the anomalous magnetic moments of the proton and neutron drives the mass shifts due to the direct interactions in the same direction. For $B > 1.5 \times 10^{14}$ T the neutron becomes stable and as the field is increased past 2.7×10^{14} T the proton becomes unstable to a decay into a neutron, positron and neutrino.

In section 2 we study the behavior of a proton, neutron and electron in an intense magnetic field, and in section 3 we study the mass shifts due to the strong forces. This effect can be calculated because the baryon masses are driven primarily by chiral condensates [6]; this would not be true for mesons as other (gluon) condensates and perturbative forces are important. Decay rates and spectrum are presented in section 4. Caveats for the validity of these results caused by the assumption of field independent magnetic moments are presented in section 5 as are limitations due to a possible vacuum instability. Conclusions and astrophysical consequences are presented in the last section.

[1] E-mail addresses: mbander@ucivmsa.bitnet; mbander@funth.ps.uci.edu.
[2] E-mail address: rub@vand.physto.se.

2. Low lying states for particles in uniform magnetic fields

The quantum mechanics of a Dirac particle with no anomalous magnetic moment in a uniform external magnetic field is straightforward. We shall present the results for the case where particles do have such anomalous moments. In reality, in fields so strong that the mass shifts induced by such fields are of the order of the mass itself one cannot define a magnetic moment as the energies are no longer linear in the external field. Schwinger [7] calculated the self-energy of an electron in an external field and we shall use his results subsequently. We cannot follow this procedure for the proton or neutron as we do not have a good field theory calculation of the magnetic moments of these particles, even for small magnetic fields; all we have at hand is a phenomenological anomalous magnetic moment. However, for fields that change the energies of these particles by only a few percent, we will consider these as point particle with the given anomalous moments. In section 5 we will discuss possible limitations of this approach.

2.1. Protons in an external field

The Dirac Hamiltonian for a proton with a uniform external magnetic field $\boldsymbol{B}$ is

$$H = \boldsymbol{\alpha}\cdot[\boldsymbol{p} - e\boldsymbol{A}(r)] + \beta M_p - \frac{e}{2M_p}(\tfrac{1}{2}g_p - 1)\beta\boldsymbol{\Sigma}\cdot\boldsymbol{B}. \tag{1}$$

The vector potential $\boldsymbol{A}(r)$ is related to the field by $\boldsymbol{A}(r) = \frac{1}{2}\boldsymbol{r}\times\boldsymbol{B}$ and $g_p = 5.58$ is the proton's Landé g-factor. We first solve this equation for the case where the momentum along the magnetic field direction is zero and then boost along that direction till we obtain the desired momentum. For $\boldsymbol{B}$ along the z direction and $p_z = 0$ the energy levels are [8]

$$E_{n,m,s} = [2eB(n+\tfrac{1}{2}) - eBs + M_p^2]^{1/2} - \frac{e}{2M_p}(\tfrac{1}{2}g_p - 1)Bs. \tag{2}$$

In the above, n denotes the Landau level, m the orbital angular momentum about the magnetic field direction and $s = \pm 1$ indicates whether the spin is along or opposed to that direction; the levels are degenerate in m. $n = 0$ and $s = +1$ yield the lowest energy:

$$E = \tilde{M}_p = M_p - \frac{e}{2M_p}(\tfrac{1}{2}g_p - 1)B. \tag{3}$$

As we shall be interested in these states only, we will drop the n and s quantum numbers. The Dirac wave function for this state is

$$\psi_{m,p_z=0}(\boldsymbol{r}) = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\phi_m(x,y), \tag{4}$$

ϕ_m's are the standard wave functions of the lowest Landau level,

$$\phi_m(x,y) = \frac{(\frac{1}{2}|eB|)^{(m+1)/2}}{\sqrt{\pi m!}}(x+\mathrm{i}y)^m \times \exp[-\tfrac{1}{4}|eB|(x^2+y^2)]. \tag{5}$$

Boosting to a finite value of p_z is straightforward; we obtain

$$E_m(p_z) = \sqrt{p_z^2 + \tilde{M}^2}, \tag{6}$$

with a wave function

$$\psi_{m,p_z}(\boldsymbol{r}) = \begin{pmatrix}\cosh\theta\\0\\\sinh\theta\\0\end{pmatrix}\frac{e^{\mathrm{i}p_z z}}{\sqrt{2\pi}}\phi_m(x,y), \tag{7}$$

where 2θ, the rapidity, is obtained from $\tanh 2\theta = p_z/E_m(p_z)$.

In the non-relativistic limit the energy becomes

$$E_m(p_z) = \tilde{M} + \frac{p_z^2}{2\tilde{M}} \tag{8}$$

and the wave function reduces to

$$\psi_{m,p_z}(\boldsymbol{r}) = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\frac{e^{\mathrm{i}p_z z}}{\sqrt{2\pi}}\phi_m(x,y). \tag{9}$$

2.2. Neutrons in an external field

For a neutron the Dirac Hamiltonian is somewhat simpler:

$$H = \boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta M_n - \frac{e}{2M_n}\frac{g_n}{2}\beta\boldsymbol{\Sigma}\cdot\boldsymbol{B}, \tag{10}$$

with $g_n = -3.82$. Again for $p_z = 0$ the states of lowest energy, the ones we shall be interested in, have energies

$$E(\boldsymbol{p}_\perp, p_z = 0) = \frac{e}{2M_n}\frac{g_n}{2}B + \sqrt{\boldsymbol{p}_\perp^2 + M_n^2}. \tag{11}$$

Boosting to a finite p_z we obtain

$$E(\boldsymbol{p}) = \sqrt{E(\boldsymbol{p}_\perp, p_z = 0)^2 + p_z^2}. \tag{12}$$

The wave functions corresponding to this energy are

$$\psi_{\boldsymbol{p}}(\boldsymbol{r}) = \frac{e^{i\boldsymbol{p}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}}\, u(\boldsymbol{p}, s = -1), \tag{13}$$

where $u(\boldsymbol{p}, s = -1)$ is the standard spinor for a particle with momentum $\boldsymbol{p}$, energy $\sqrt{\boldsymbol{p}^2 + M_n^2}$ (not the energy of eq. (12)) and spin down.

In the non-relativistic limit

$$E(\boldsymbol{p}) = M_n + \frac{e}{2M_n}\frac{g_n}{2}B + \frac{\boldsymbol{p}^2}{2M_n}, \tag{14}$$

and the wave functions are

$$\psi_{\boldsymbol{p}}(\boldsymbol{r}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \frac{e^{i\boldsymbol{p}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}}. \tag{15}$$

2.3. *Electrons in an external field*

We might be tempted to use, for the electron, the formalism used for the proton with the Landé factor replaced by $g_e = 2 + \alpha/\pi$. However, as we shall see for magnetic fields sufficiently strong as to make the proton heavier than the neutron, the change in energy of the electron would appear to be larger than the mass of the electron itself. The point particle formalism breaks down and we have to solve QED, to one loop, in a strong magnetic field; fortunately this problem was treated by Schwinger [7]. The energy of an electron with $p_z = 0$, spin up and in the lowest Landau level is

$$E_{m,p_z=0} = M_e\left[1 + \frac{\alpha}{2\pi}\ln\left(\frac{2eB}{M_e^2}\right)\right]. \tag{16}$$

For field strengths of subsequent interest this correction is negligible; the energy of an electron in the lowest Landau level, with spin down and a momentum of $p_z \boldsymbol{z}$ is

$$E_{m,p_z} = \sqrt{p_z^2 + M_e^2}, \tag{17}$$

and with wave function similar to those of the proton,

$$\psi_{m,p_z}(\boldsymbol{r}) = \begin{pmatrix} 0 \\ \cosh\theta \\ 0 \\ \sinh\theta \end{pmatrix} \frac{e^{ip_z z}}{\sqrt{2\pi}}\, \phi_m^*(x, y), \tag{18}$$

where the boost rapidity, 2θ, is defined below eq. (7) while the Landau level wave function is defined in eq. (5). The reason the complex conjugate wave function appears is that the electron charge is opposite to that of the proton.

3. Effects of magnetic fields on strong forces

We must be sure that shifts due to changes of color strong forces will not shift states in the opposite direction. The best method to study masses of QCD bound states is the use of sum rules [6]. This method uses the SVZ [9] generalized short distance expansion that includes not only perturbative pieces, but also higher dimensional operators like the chiral and gluon condensates reflecting the non-abelian nature of the vacuum. Fortunately the proton has a simple structure [6] which reflects the fact that if chiral symmetry is restored the proton and neutron masses vanish:

$$M_{p,n} = 3a\langle q\bar{q}\rangle^{1/3} + \text{small corrections}, \tag{19}$$

where a is a constant. Meson mass terms are more involved; for example the ρ mass is

$$M_\rho = b(\text{perturbative terms}) + c\langle G_{\mu\nu}G^{\mu\nu}\rangle + d\langle q\bar{q}\rangle. \tag{20}$$

b, c and d are constants of comparable magnitude [6]. As we shall show it is only the change of $\langle q\bar{q}\rangle$ due to external magnetic fields that may be obtained in a reliable manner.

In the presence of external fields we expect the chiral condensates for quarks of different charges to vary, and eq. (19) becomes

$$M_p^3 = a(2\langle u\bar{u}\rangle + \langle d\bar{d}\rangle),$$
$$M_n^3 = a(2\langle d\bar{d}\rangle + \langle u\bar{u}\rangle). \tag{21}$$

To first order in condensate changes we find

$$\delta M_p = \frac{M_p}{9}\left(2\frac{\delta\langle u\bar{u}\rangle}{\langle u\bar{u}\rangle} + \frac{\delta\langle d\bar{d}\rangle}{\langle d\bar{d}\rangle}\right),$$

$$\delta M_n = \frac{M_n}{9}\left(2\frac{\delta\langle d\bar{d}\rangle}{\langle d\bar{d}\rangle} + \frac{\delta\langle u\bar{u}\rangle}{\langle u\bar{u}\rangle}\right). \quad (22)$$

Combining:

$$\delta M_p - \delta M_n = \frac{M}{9}\left(\frac{\delta\langle u\bar{u}\rangle}{\langle u\bar{u}\rangle} - \frac{\delta\langle d\bar{d}\rangle}{\langle d\bar{d}\rangle}\right). \quad (23)$$

A simple method for studying the behavior of the chiral condensates in the presence of external constant fields is through the use of the Nambu–Jona-Lasinio model [10]. This has been done by Klevansky and Lemmer [11] and a fit to their results is

$$\langle q\bar{q}\rangle(B) = \langle q\bar{q}\rangle(0)\left[1 + \left(\frac{e_q B}{\Lambda^2}\right)^2\right]^{1/2}, \quad (24)$$

with $\Lambda = 270$ MeV and e_q the charge on the quark. To lowest order we find

$$\frac{\delta\langle q\bar{q}\rangle}{\langle q\bar{q}\rangle} = \frac{1}{2}\left(\frac{e_q B}{\Lambda^2}\right)^2 \quad (25)$$

and

$$\delta M_p - \delta M_n = \frac{M}{54}\left(\frac{eB}{\Lambda^2}\right)^2. \quad (26)$$

As in the previous section, these corrections are such as to drive the proton energy up faster than that of the neutron. One can understand the sign of this effect; the radius of a quark–antiquark pair will decrease with increasing magnetic field thus making the condensate larger. As the u quark has twice the charge of the d quark, its condensate will grow faster and as there are more u quarks in the proton than in the neutron its mass will increase faster.

The fact that our estimate of the sign of the neutron–proton mass difference is the same as that due to electromagnetic effects is crucial. QCD sum rules and our method of evaluating the chiral condensates are both crude and the magnitude of the mass difference is uncertain. Had the sign of the hadronic correction been opposite, cancellations could have occurred and the argument for a narrowing of the mass difference and ultimate reversal could not have been made.

4. Proton life time

4.1. Decay kinematics

Combining eq. (3), eq. (14) and eq. (26) we find the proton–neutron energy difference as a function of the applied magnetic field:

$$\Delta(B) = -1.3 + 0.38B_{14} + 0.11B_{14}^2 \text{ MeV}. \quad (27)$$

B_{14} is the strength of the magnetic field in units of 10^{14} T. The neutron becomes stable for $B > 1.5 \times 10^{14}$ T and the proton becomes unstable to β decay for $B > 2.7 \times 10^{14}$ T. We shall now turn to a calculation of the life time of the proton for fields satisfying the last inequality.

4.2. Proton, neutron and electron fields

With the wave functions of the various particles in the magnetic fields we may define field operators for these particles. For the proton and electron we shall restrict the summation over states to the lowest Landau levels with spin up, down respectively; for magnetic fields of interest the other states will not contribute to the calculation of decay properties. For the same reason, the neutron field will be restricted to spin down only. The proton and neutron kinematics will be taken as non-relativistic,

$$\Psi_p(\boldsymbol{r}) = \sum_m \int \mathrm{d}p_z \left[a_m(p_z)\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\frac{\mathrm{e}^{\mathrm{i}p_z z}}{\sqrt{2\pi}}\phi_m(x,y) + b_m^\dagger(p_z)\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\frac{\mathrm{e}^{-\mathrm{i}p_z z}}{\sqrt{2\pi}}\phi_m(x,y)\right], \quad (28)$$

with $\phi_m(x,y)$ defined in eq. (5) and the energy, $E_m(p_z)$ in eq. (8). $a_m(p_z)$ is the annihilation operator for a proton with momentum $p_z z$ and angular momentum m; $b_m(p_z)$ is the same for the negative energy states. For the neutron the field is

$$\Psi_n(\boldsymbol{r}) = \int \mathrm{d}^3p \left[a(\boldsymbol{p}) \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \frac{\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}} + b^\dagger(\boldsymbol{p}) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \frac{\mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}} \right], \tag{29}$$

with an obvious definition of the annihilation operators. For the electron we use fully relativistic kinematics and the field is

$$\Psi_e(\boldsymbol{r}) = \sum_m \int \mathrm{d}p_z \sqrt{\frac{M_e}{E}} \times \left[a_m(p_z) \begin{pmatrix} 0 \\ \cosh\theta \\ 0 \\ \sinh\theta \end{pmatrix} \frac{\mathrm{e}^{\mathrm{i}p_z z}}{\sqrt{2\pi}} \phi_m^*(x,y) + b_m^\dagger(p_z) \begin{pmatrix} 0 \\ \cosh\theta \\ 0 \\ \sinh\theta \end{pmatrix} \frac{\mathrm{e}^{-\mathrm{i}p_z z}}{\sqrt{2\pi}} \phi_m^*(x,y) \right]. \tag{30}$$

4.3. Decay rates and spectrum

The part of the weak Hamiltonian responsible for the decay $p \to n + e^+ + \nu_e$ is

$$H = \frac{G_F}{\sqrt{2}} \int \mathrm{d}^3x\, \overline{\Psi}_n \gamma_\mu (1+\gamma_5) \Psi_p \overline{\Psi}_\nu \gamma^\mu (1+\gamma_5) \Psi_e. \tag{31}$$

For non-relativistic heavy particles the matrix element of this Hamiltonian between a proton with quantum numbers $p_z = 0, m = m_i$, a neutron with momentum $\boldsymbol{p}_n$, a neutrino with momentum $\boldsymbol{p}_\nu$ and an electron in state $m = m_f$ and with $p_{z,e}$ is

$$\langle H \rangle = \frac{2G_F}{(2\pi)^3} \left(\frac{E_e + p_{z,e}}{E_e - p_{z,e}} \right)^{1/4} \sin(\theta_\nu/2) \sqrt{\frac{M_e}{E_e}} \times \delta(p_{z,e} + p_{z,\nu} + p_{z,n}) \times \int \mathrm{d}x\, \mathrm{d}y\, \phi_{m_f}^*(x,y) \phi_{m_i}(x,y) \times \exp[-\mathrm{i}(\boldsymbol{p}_{\perp,n} + \boldsymbol{p}_{\perp,\nu}) \cdot \boldsymbol{r}_\perp]; \tag{32}$$

θ_ν is the azimuthal angle of the neutrino. The integral in the above expression can be evaluated in a multipole expansion. Note that the natural extent of the integral in the transverse direction is $1/\sqrt{eB}$ whereas the neutron momenta are, from eq. (27), of the order of $\sqrt{eB(0.12 + 0.04B_{14})}$; thus setting the exponential term in this integral equal to one will yield a good estimate for the rate and spectrum of this decay. The positron spectrum is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}p_{z,e}} = \frac{4}{3} \frac{G_F^2 M_p}{(2\pi)^6} \frac{E_e + p_{z,e}}{E_e} (\Delta - E_e)^3, \tag{33}$$

where Δ is defined in eq. (27). For $\Delta \gg M_e$ the total rate is easily obtained:

$$\Gamma = \frac{2}{3} \frac{G_F^2 M_p}{(2\pi)^6} \Delta^4. \tag{34}$$

For $B = 5 \times 10^{14}$ T, the lifetime is $\tau = 6$ s.

5. Caveats and limitations

For general magnetic fields we expect the masses of particles to be non-linear functions of these fields. Such an expression has been obtained, to order α, for the electron [7]. For small fields this reduces to a power series, up to logarithmic terms, in B/B_c, where B_c is some scale. For the electron $B_c = M_e^2/e$. For the hadronic case the value of B_0 is uncertain. $B_c = M_p^2/e = 1.7 \times 10^{16}$ T is probably too large and M_p should be replaced by a quark constituent mass and e be e_q; in that case $B_c = (2\text{–}4) \times 10^{15}$ T, depending on the quark type. This is also the range of values of Λ^2/e_q in eq. (24). The effects we have studied need fields around a few $\times 10^{14}$ T or an order of magnitude smaller than the lowest candidate for B_c. Eq. (27) may be viewed as a power series expansion up to terms of order $(B/B_c)^2$; as the coefficient of the quadratic term was obtained from a fit to a numerical solution, logarithms of B/B_c may be hidden in the coefficient. As in ref. [7], even powers will be spin independent and the odd ones will be linear in the spin direction and may be viewed as field dependent corrections to the magnetic moment. We cannot prove, but only hope, that the coefficient of the $(B/B_c)^3$ term, the first correction to the magnetic moment is not unusually large; should it turn out to be big and of opposite sign to the linear and quadratic terms, the conclusions of this work would be invalidated. These arguments, probably, apply best to the field dependence of the magnetic moments of the quarks rather than the total moment of the baryons. We may ask what is the effect on these magnetic moments due to changes in the "orbital" part of the quark wave functions. To first order we expect no effect as all the quarks are in S states and there is no orbital contribution to the total moment. The next order perturbation correction will be *down* by $(r_b/r_c)^4$ compared to the leading effect; r_b is the hadronic radius and r_c is the quark's cyclotron radius. This again contributes to the $(B/B_c)^3$ term in the expansion for the energy of a baryon.

An other limitation is due to the results of ref. [5] where we have shown that fields of the order of a few $\times 10^{14}$ T are screened by changes in chiral condensates. In fact, as the chiral condensate will, in large fields, point in the charged π direction, the baryonic states will not have a definite charge. Whether the proton–neutron reversal takes place for fields below those that are screened by chiral condensates or vice versa is a subtle question; the approximations used in this paper and in ref. [5] are not reliable to give an unambiguous answer. The treatment of the effects of magnetic fields on the strong force contributions to the baryon masses relies on the Nambu–Jona-Lasinio model, and in ref. [5] the variation of f_π with magnetic field was not taken into account. We hope to return to these questions in a future work. It is clear from this discussion and from the one in the previous section that we cannot push the results of this paper past a few $\times 10^{14}$ T.

6. Conclusions and experimental consequences

We studied the mass evolution of protons, neutrons and electrons in magnetic fields and concluded that due to electromagnetism alone, a proton will decay in a very intense field. Including the effects of chiral condensates diminishes the field even further. Qualitatively, it is clear that the effect enhances the electromagnetic contribution but its exact value depends on the model. This points to a novel astrophysical mechanism for creation of extra galactic positrons. There is indeed an overabundance of positrons as compared with existing mechanisms [12]. As the magnetic moment of leptons is greatly reduced in such fields [7] the extraction mechanism for positrons from such fields [2] must be recalculated.

Acknowledgement

H.R.R. wishes to thank the hospitality of the University of California, Irvine and of the Hebrew University, Jerusalem. M.B. wishes to thank the hospitality of the Weizmann Institute. This work was supported in part by a National Science Foundation Grant PHY-9208386 and EEC SCIENCE Grants and the Swedish National Research Council.

References

[1] J. Ambjørn and P. Olesen, Nucl. Phys. B 315 (1989) 606; Niels Bohr preprint NBI-HE-93-17.

[2] V. Berezinsky and H.R. Rubinstein, Nucl. Phys. B 323 (1989) 95.

[3] R. Narayan, B. Paczyński and T. Piran, Harvard-Smithsonian Center of Astrophysics preprint CFA-3372, submitted to Astrophys. J. Lett.;
T. Piran, in: Proc. 26th Intern. Conf. on High energy physics (August 1992), to appear.

[4] M. Bander and H.R. Rubinstein, Phys. Lett. B 280 (1992) 121.

[5] M. Bander and H.R. Rubinstein, Phys. Lett. 289 (1992) 385.

[6] H. Reinders, H.R. Rubinstein and S. Yazaki, Phys. Rep. 127 (1985) 1.

[7] J. Schwinger, Particles, sources, and fields, Vol. 3 Advanced Book Program (Addison-Wesley, Redwood City, CA, 1988) p. 164.

[8] C. Itzykson and J.-B. Zuber, Quantum field theory (McGraw Hill, New York, 1980) p. 67.

[9] M.A. Shifman, A.J. Vainstein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385; 448.

[10] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; 124 (1961) 246.

[11] S.P. Klevansky and R.H. Lemmer, Phys. Rev. D 39 (1989) 3478.

[12] C.J. Cesarsky, private communication;
W.R. Webber, in: 20th Intern. Cosmic ray Conf., Vol. 8 (Nauka, Moscow, 1988) p. 65.